SUMMATIVE ASSESSMENT – II, 2015, MATHEMATICS, Class – IX

SOLVED SAMPLE QUESTION PAPER

JST201504

Time allowed: 3 hours **Maximum Marks: 90**

General Instructions :

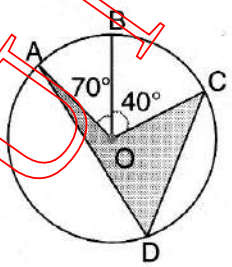
1. **All** questions are compulsory.
2. The question paper consists of **31 questions** divided into four **sections A, B, C** and **D**. **Section 'A'** comprises of **4 questions** of **1 mark** each, **Section 'B'** comprises of **6 questions** of **2 marks** each, **Section 'C'** comprises of **10 questions** of **3 marks** each and **Section 'D'** comprises of **11 questions** of **4 marks** each.
3. There is no overall choice.
4. Use of calculator is not permitted.

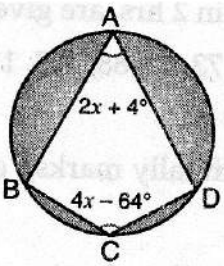
SECTION — A

1. Find the co-ordinate of the point where lines $ax = by$ and $ay = bx$ intersects each other.
2. Find the value of C if the line $2x + 3y + C = 0$ passes through origin.
3. Following observations have been written in ascending order. If median of the data is 22, then find the value of x.
   11, 12, 14, 16, 18, $x + 2$, $x + 4$, 30, 32, 35, 41.
4. If $PQRS$ is a parallelogram, then find the value of $\angle Q + \angle R$ and $\angle R + \angle S$.

SECTION — B

5. There are 100 students in a class. 40 of them are girls. The average marks of the boys in science is 75% and that of the girls is 65%. find the average marks of the class in science.
6. Find the total surface area of a solid cone if its slant height is 21 cm and diameter of its base is 24 cm.
7. The class-marks of classes in a distribution are 6, 10, 14, 18, 22, 26, 30. Find :
   (a) class size (b) lower limit of second class
   (c) upper limit of last class (d) third class .
8. What is the volume of a cube whose total surface area is 864 m^2 ?
9. In the given figure, A, B and C are three points on a circle with centre O such that $\angle BOC = 40°$ and $\angle AOB = 70°$. If D is a point on the circle, find $\angle ADC$.





Q-10

10. In the figure, find the value of x.

11. Draw the graph of a linear equation $y = mx + c$ for $m = 2$ and $c = 1$. Read from the graph the value of y when $x = \frac{3}{2}$.

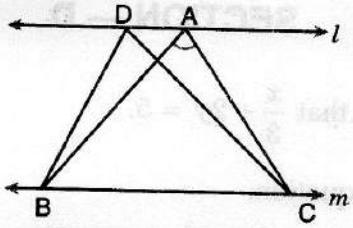
12. The linear equation that converts Fahrenheit 'F' to Celsius 'C' is given by the equation $C = (5F - 160°)/9$.

(i) If the temperature is 104°F, what is the temperature in Celsius ?

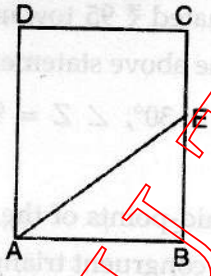
(ii) If the temperature is 35°C, what is the temperature in Fahrenheit ?

(iii) Is there a temperature numerically same in both Fahrenheit and Celsius ? If yes, find it.

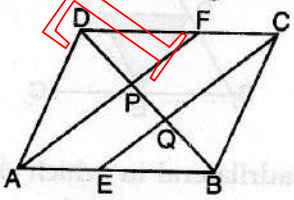
13. In the given figure, ABC and DBC are triangles on the same base and between parallel lines. If AB = 3 cm, BC = 5 cm, $\angle A = 90°$, find area ΔDBC.



14. $ABCD$ is a parallelogram. If E is the mid-point of BC and AE is the bisector of $\angle A$. Prove that $AB = \frac{1}{2} AD$.



15. E and F are mid-points of sides AB and CD respectively of a parallelogram $ABCD$. AF and CE intersect diagonal BD in P and Q respectively. Prove that diagonal BD is trisected at P and Q.



16. How many square metre of canvas is required for making a conical tent whose height is 3.5 m and the radius of the base is 12 m ? $\left(\text{Use } \pi = \frac{22}{7}\right)$

17. The volume of a sphere is $905\frac{1}{7}$ cm^3. Determine its diameter and its surface area. $\left(\text{Use } \pi = \frac{22}{7}\right)$

18. The distance (in km.) covered by 30 cars in 2 hrs. are given below.

125, 107, 120, 90, 84, 100, 56, 140, 93, 149, 73, 68, 88, 135, 115, 120, 90, 120, 136, 144, 104, 135, 60, 79, 83, 100, 76, 91, 89, 120.

From a frequency distribution table (with tally marks), one of the intervals being 100 -120. (120 not included)

19. A die is thrown 500 times and the frequencies of outcome of the number 1, 2, 3, 4, 5 and 6 are given below :

Outcome	1	2	3	4	5	6
Frequency	75	85	90	60	80	110

Find the probability of happening of each of the events. Also find the sum of all probabilities.

20. On a busy road, following data was observed about cars passing through it and number of occupants :

No. of occupants	1	2	3	4	5
No. of cars	29	26	23	17	5

Find the chance that it has :

(i) exactly 5 occupants.

(ii) more than 2 occupants.

(iii) less than 5 occupants.

21. (a) Express y in terms of x given that $\frac{x}{3} + 2y = 5$.

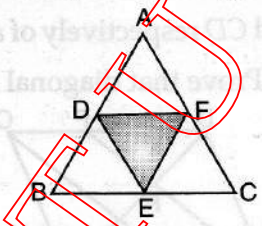
(b) Plot the graph of the above equation.

(c) Check whether (3, 2) is a solution of the given equation.

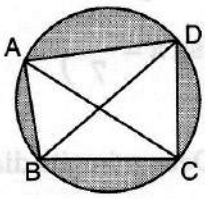
22. Mahesh and Altaf, two students donated ₹ 95 towards the Prime Minister's Relief Fund. Write a linear equation in two variables for the above statement. Also draw the graph for the same.

23. Construct a ΔXYZ in which $\angle Y = 30°$, $\angle Z = 90°$ and the perimeter of a triangle XYZ is 18 cm.

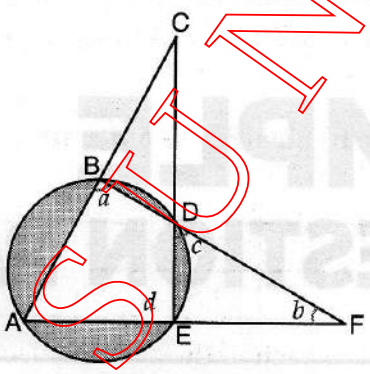
24. In the given figure, D, E and F are mid-points of the sides AB, BC and CA respectively of ΔABC. Show that ΔABC is divided into four congruent triangles by joining the points D, E and F.

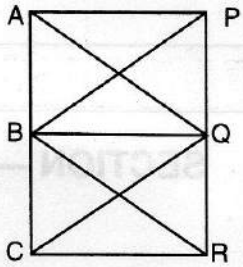


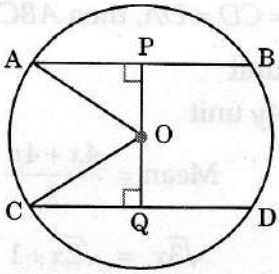
25. In the figure, $ABCD$ is a cyclic quadrilateral in which AC and BD are diagonal. If $\angle ABC = 55°$ and $\angle BAC = 45°$, find $\angle BCA$.



26. In the given figure, $\angle BCD = 43°$ and $\angle BAE = 62°$. Find the value of a, b, c, d.







27. In the given figure, $AP \parallel BQ \parallel CR$. Prove that ar (AQC) = ar (PBR).

28. In figure, O is the centre of the circle of radius 5 cm, $OP \perp AB$, $OQ \perp CD$, $AB \parallel CD$, AB = 6 cm, CD = 8 cm. Determine PQ.

29. A circus tent is in the form of a cone of height 15 m and diameter 16 m. Find the length of the canvas needed to make the tent if the width of the canvas 2 m. (use $\pi = 3.14$)

30. $\frac{3}{4}$ th of cylindrical can contains milk. The height of the can is 1.4 m and radius is 0.4 m. This milk is poured into small cylindrical glasses of height 10 cm and radius 5 cm. How many small glasses are needed to empty the can ?

31. Draw histogram and frequency polygon for the following data :

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 –70	70 – 80
No. of candidates	2	5	6	4	8	10	5

(i) Which mathematical concept is used in the above problem ?
(ii) What is its value ?

Solutions

1. $ax = by$ ...(i)

$ay = bx$

$y = \frac{bx}{a}$ ...(ii)

From equations (i) and (ii), $ax = b \times \frac{bx}{a}$

$a^2x = b^2x$

$x(a^2 - b^2) = 0$

$x = \frac{0}{a^2 - b^2} = 0$

From (ii), we get $y = \frac{b}{a} \times 0 = 0$

Hence, intersecting point is (0, 0). **1**

2. $2x + 3y + c = 0$

The line passes through origin (0, 0)

$\therefore \quad 2 \times 0 + 3 \times 0 + c = 0$

$0 + 0 + c = 0$

$\boxed{c = 0}$ **1**

3. **Data :** 11, 12, 14, 16, 18, $x + 2$, $x + 4$, 30, 32, 35, 41

$N = 11$ (odd)

Median $= \left(\frac{N+1}{2}\right)^{th}$ term $\quad 22 = \left(\frac{11+1}{2}\right)^{th}$ term

$22 = 6^{th}$ term $\quad 22 = x + 2 \quad x = 22 - 2 = 20$

4. In || gm, the sum of adjacent angles is 180°

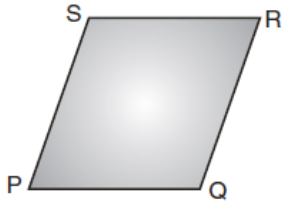
$\therefore \quad \angle Q + \angle R = 180°$ ...(i) ½

and opposite angles are equal.

$\therefore \quad \angle Q = \angle S$

From (i), we get

$\angle S + \angle R = 180°$. ½



SECTION — 'B'

5. Girls = 40, Boys = 100 – 40 = 60

Total marks for boys = 60 × 75 = 4500

Total marks for girls = 40 × 65 = 2600 **1**

Sum of class = 4500 + 2600 = 7100

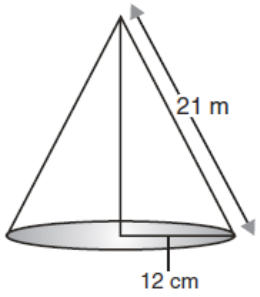
Mean marks of the class = $\frac{7100}{100}$ = 71% **1**

6. Radius of cone = $\frac{24}{2}$ = 12 cm

Total surface area of cone = $\pi rl + \pi r^2$ **1**

$= \pi r(l + r) = \frac{22}{7} \times 12\,(21 + 12)$

$= \frac{22}{7} \times 12 \times 33 = 1244{\cdot}57\ \text{cm}^2$ **1**



7. Class interval are 4 – 8, 8 – 12, 12 – 16, 16 – 20, 20 – 24, 24 – 28, 28 – 32

(a) class size = 8 – 4 = 4

(b) lower limit of second class = 8

(c) upper limit of last class = 32

(d) third class = 12 – 16 (½ + ½ + ½ + ½)

8. Let the side of cube be a.

Then total surface area of cube = $6a^2 = 864$

$a^2 = 144$

$a = 12$ **1**

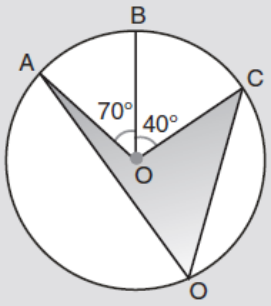
Volume of cube = $a^3 = (12)^3 = 1728\ \text{m}^3$. **1**

9. Given, A, B and C are three points on a circle with centre O, such that $\angle BOC = 40°$ and $\angle AOB = 70°$.

Here, $\angle AOC = \angle AOB + \angle BOC$.

$= 70° + 40° = 110°$

$\therefore$ Arc ABC makes 110° at the centre of the circle.



Then $\angle ADC = \frac{1}{2} \angle AOC$ **1**

(the angle subtended by an arc at the centre is double the angle subtended by it at any point of the circle)

$= \frac{1}{2} \times 110° = 55°$

$\therefore \quad \angle ADC = 55°$ **1**

10. In a cyclic quadrilateral

$$2x + 4° + 4x - 64 = 180°$$
$$6x - 60 = 180°$$ **1**
$$6x = 240°$$
$$x = \frac{240°}{6}$$
$$x = 40°$$ **1**

SECTION — C

11. Equation $y = mx + c$

Put $m = 2$ and $c = 1$ $\quad y = 2x + 1$ ...(i)

Put $x = 1$ in equation (i) $\quad y = 2 \times 1 + 1 = 3$

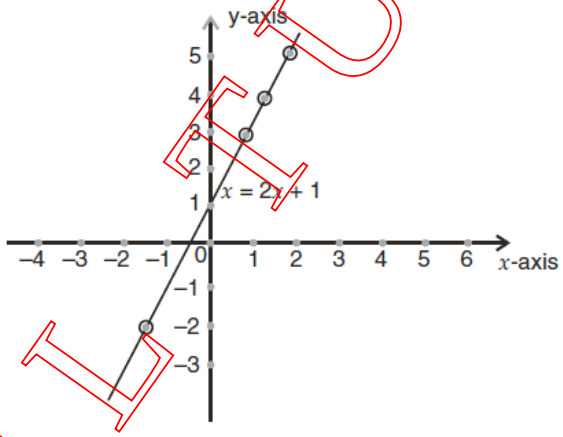
Put $x = 2$ in equation (i) $\quad y = 2 \times 2 + 1 = 5$

Put $x = \frac{3}{2}$ in equation (i) $\quad y = 2 \times \frac{3}{2} + 1 = 4$

Put $x = \frac{-3}{2}$ in equation (i) $\quad y = 2 \times \frac{-3}{2} + 1$ **1**

$= -2$ **1½**

x	1	2	$\frac{3}{2}$	$\frac{-3}{2}$
y	3	5	4	–2



Value of y, when $x = \frac{3}{2}$ is 4. **½**

12. $C = \frac{(5F - 160°)}{9}$ ...(i)

(i) If temperature = 104°F

From (i), $C = \frac{5 \times 104 - 160}{9} = \frac{520 - 160}{9}$

$C = \frac{360°}{9} = 40°$ **1**

(ii) Temperature = 35°C

from (i) $35° = \frac{5F - 160}{9}$

$315 = 5F - 60$

$5F = 375°$

$F° = 75°$ **1**

Put the same temperature x for both temperatures.

From (i), $x^\circ = \dfrac{5x - 160^\circ}{9}$

$9x = 5x - 160^\circ$

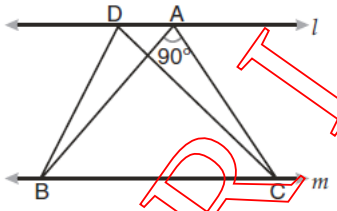
$9x - 5x = -160^\circ$

$4x = -160^\circ$

$x = -40^\circ$

Yes, it is (-40°). **1**

13. Here $AB = 3$ cm, $BC = 5$ cm, $\angle A = 90^\circ$



In right angled triangle ABC

$BC^2 = AB^2 + AC^2$

$(5)^2 = 3^2 + AC^2$

$AC^2 = 25 - 9 = 16$

$AC = 4$ cm **1**

ΔABC and ΔDBC are on the same base BC and between same parallels l and m. **1**

$\therefore$ ar (DBC) = ar (ABC)

$= \dfrac{1}{2} \times$ base $\times$ height

$= \dfrac{1}{2} \times AB \times AC$

$= \dfrac{1}{2} \times 3 \times 4$

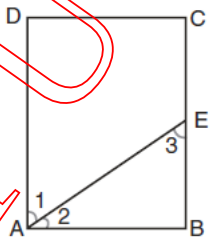
$= 6$ cm^2. **1**

14. Here $\angle 1 = \angle 2$ (AE is angle bisector)

But $\angle 1 = \angle 3$ (Alternate angle as $AD \parallel BC$)

$\therefore$ $\angle 3 = \angle 2$ **1**

Hence, $BE = AB$ (sides opposite to equal angles)



But $BE = \dfrac{1}{2} BC$, (E is the mid-point of BC)

$\therefore$ $AB = \dfrac{1}{2} BC$ **1**

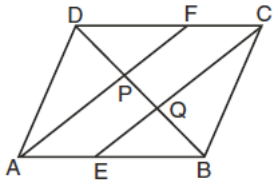
and $BC = AD$ (opposite sides of $\parallel$ gm)

$\therefore$ $AB = \dfrac{1}{2} AD$ **1**

15. According to question, E and F the mid-points of sides AB and CD.

$\because$ $AE = \dfrac{1}{2} AB$

and $CF = \dfrac{1}{2} CD$



$\because$ In the parallelogram opposite sides are equal, so

$AB = CD$

$\therefore$ $AE = CF$ **1**

Again, $AB \parallel CD$

So, $AE \parallel FC$

Hence, $AECF$ is a parallelogram

In ΔABP,

E is the mid-point of AB

$EQ \parallel AP$ **1**

$\therefore$ Q is mid-point of BP

Similarly, P is the mid-point of DQ.

$$DP = PQ = QB$$

$\therefore$ Line segments AF and EC trisect the diagonal BD. 1

16. Height of cone (h) = 3·5 m

Radius of base (r) = 12 m

$$\text{Slant height } (l) = \sqrt{h^2 - r^2} = \sqrt{(3{\cdot}5)^2 + (12)^2}$$

$$= \sqrt{12{\cdot}25^2 + 144^2}$$

$$= \sqrt{156{\cdot}25} = 12{\cdot}5 \text{ cm} \quad 1$$

Area of canvas for making a conical tent

$$= \pi rl$$

$$= \frac{22}{7} \times 12 \times 12{\cdot}5$$

$$= 471{\cdot}43 \text{ cm}^2 \text{ (app.)} \quad 1$$

17. Volume of sphere $= \frac{4}{3}\pi r^3$

$$905\frac{1}{7} = \frac{4}{3} \times \frac{22}{7} \times r^3$$

$$r^3 = \frac{6336}{7} \times \frac{7 \times 3}{4 \times 22} = \frac{288 \times 3}{4}$$

$$r^3 = 216 = (6)^3$$

$$r = 6 \text{ cm} \quad 1\frac{1}{2}$$

Diameter of sphere $= 2r = 2 \times 6 = 12$ cm ½

$$\text{Surface Area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 6 \times 6$$

$$= \frac{3168}{7} = 452\frac{7}{7} \text{ cm}^2 \quad 1$$

18.

Distance (in km)	Tally Marks	Frequency
40 — 60	|	1
60 — 80	𝍸	5
80 — 100	𝍸 III	8
100 — 120	𝍸	5
120 — 140	𝍸 III	8
140 — 160	III	3

3

19. Total outcomes = 75 + 85 + 90 + 60 + 80 + 110

= 500

$$\text{P(getting 1)} = \frac{\text{Number of outcomes}}{\text{Total number of possibilities}}$$

$$= \frac{75}{500} = \frac{3}{20}$$

$$\text{P(getting 2)} = \frac{85}{500} = \frac{17}{100}$$

$$\text{P(getting 3)} = \frac{90}{500} = \frac{9}{50}$$

22. Let Mahesh donated the money = ₹ x
All of denoted the money = ₹ y

According to question,

$$x + y = 95$$
$$y = 95 - x \quad \ldots(i)$$

Put $x = 20$ in equation (i)

$$y = 95 - 20 = 75$$

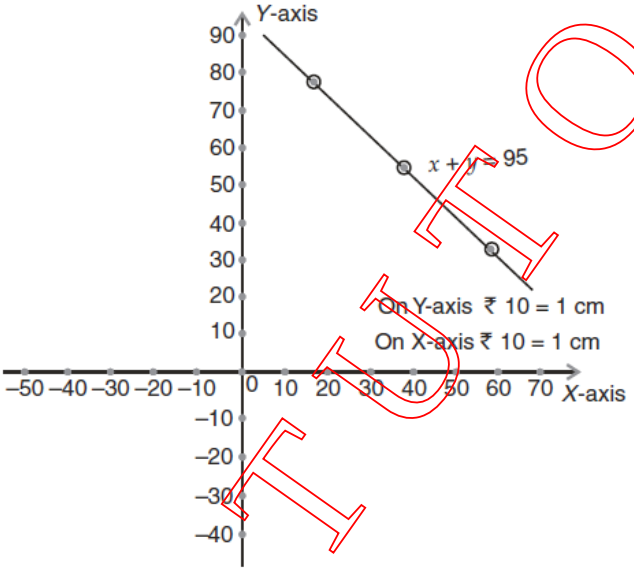
Put $x = 40$ in equation (i)

$$y = 95 - 40 = 55$$

Put $x = 60$ in equation (i)

$$y = 95 - 60 = 35$$ **2**

x	20	40	60
y	75	55	35



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23. Steps of Construction : **2**

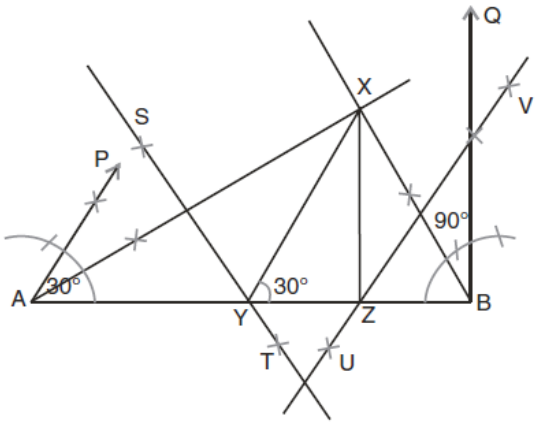
(1) Draw a line segment AB = 18 cm ($XY + YZ + ZX$ = 18 cm)

(2) Construct an angle $\angle PAB = 30°$ at point A and an angle $\angle QBA = 90°$ at point B.

(3) Bisect $\angle PAB$ and $\angle QBA$. These bisectors inter sect each other at point X.

(4) Draw perpendicular bisector ST of AX and UV of BX.

(5) Perpendicular bisector ST intersect AB at Y and UV intersect AB at Z. Join XY, XZ, then ΔXYZ is the required triangle.



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24. D and E are mid-points of AB and BC respectively.

$\therefore$ $DE \parallel AC$ **1**

Similarly, $DF \parallel BC$ and $EF \parallel AB$

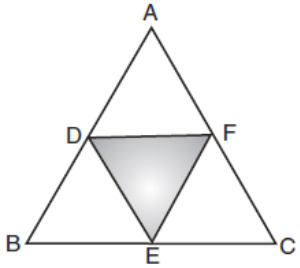
$\therefore$ $ADEF$, $BDEF$ and $DFCE$ are all parallelogram **1**

DE is the diagonal of parallelogram $BDFE$.

$\therefore$ $\Delta BDE \cong \Delta FED$ **1**

and $\Delta EFC \cong \Delta FED$

$\therefore$ All four triangles are concurrent. **1**



$$P(\text{getting } 4) = \frac{60}{500} = \frac{3}{25}$$

$$P(\text{getting } 5) = \frac{80}{500} = \frac{4}{25}$$

$$P(\text{getting } 6) = \frac{110}{500} = \frac{11}{50} \quad \textbf{2}$$

$$\text{Sum of all possibilities} = \frac{3}{20} + \frac{17}{100} + \frac{9}{50} + \frac{3}{25} + \frac{4}{25} + \frac{11}{50}$$

$$= \frac{15+17+18+12+16+22}{100} = \frac{100}{100} = 1 \quad \textbf{1}$$

20. Total no. of cars = 100

(i) P (exactly 5 occupants) $= \frac{5}{100} = \frac{1}{20}$

(ii) P (more than 2 occupants) $= \frac{23+17+5}{100} = \frac{45}{100} = \frac{9}{20}$

(iii) P (less than 5 occupants) $= \frac{29+26+23+17}{100} = \frac{95}{100} = \frac{19}{20}$ **(1 + 1 + 1)**

SECTION — 'D'

21. (a) $\frac{x}{3} + 2y = 5$

$$2y = 5 - \frac{x}{3} = \frac{15-x}{3}$$

$$y = \frac{15-x}{6} \quad \text{... (i)}$$

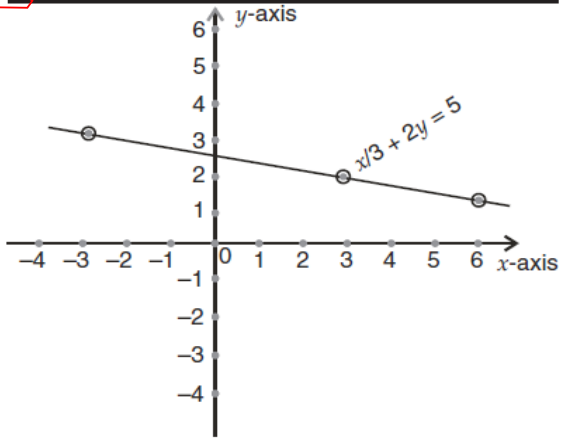
is the expression of y in term of x ½

(b) Put $x = 3$ in equation (i) $y = \frac{15-3}{6} = 2$ **2**

Put $x = 6$ in equation (i) $y = \frac{15-6}{6} = \frac{9}{6} = 1{\cdot}5$

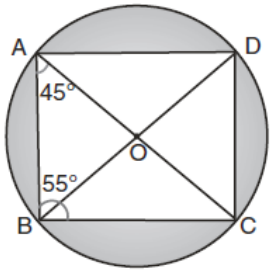
Put $x = -3$ in equation (i) $y = \frac{15-(-3)}{6} = \frac{15+3}{6} = 3$ **1**

x	3	6	–3
y	2	1	3



(c) Yes, (3, 2) is a solution of the given equation. ½

25. In ΔABC,



$\angle BAC + \angle ABC + \angle BCA = 180°$ (Angle sum property) **2**

$45° + 55° + \angle BCA = 180°$

$\angle BCA = 180° - 100°$ **2**

$= 80°$

26.

$\angle BCD = 43°$ and $\angle BAE = 62°$

In ΔACE,

$43° + 62° + d = 180°$

$d = 180° - 105°$

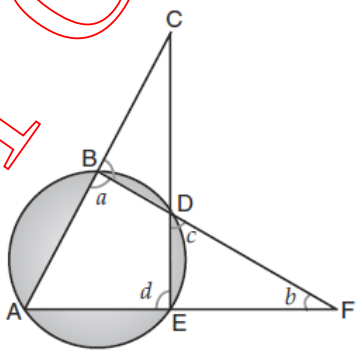
$\boxed{d = 75°}$ **1**

$a + d = 180°$ (opp. angles of cyclic quad. are supplementary)

$a + 75° = 180°$ **1**

$\boxed{a = 105°}$

In ΔABF,



$62° + 105° + b = 180°$

$b = 180° - 167°$ **1**

$\boxed{b = 13°}$

$\angle DEF = 180° - 75° = 105°$

In ΔDEF,

$105° + 13° + c = 180°$

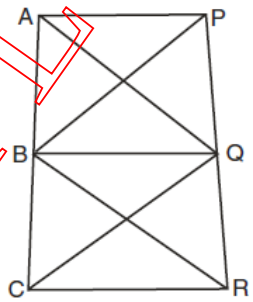
$118° + c = 180°$

$c = 180° - 118°$

$c = 62°$ **1**

$a = 105°, b = 13°, c = 62°, d = 75°$

27.



Given, $BQ \| CR$

Therefore, ΔBCQ and ΔBQR are on the same base and between the same parallels BQ and CR

So, $\text{ar}(\Delta BCQ) = \text{ar}(\Delta BQR)$ ...(i) **1½**

Also, $AP \| BQ$ (given)

Therefore, ΔABQ and ΔPBQ are on the same base BQ and between same parallels BQ and AP

$\therefore$ $\text{ar}(\Delta ABQ) = \text{ar}(\Delta PBQ)$ ...(ii) **1½**

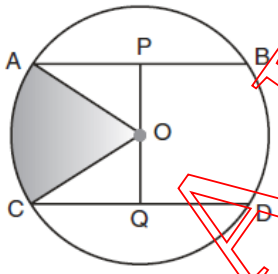
Adding (i) and (ii), we get

$(\Delta BCQ) + \text{ar}(\Delta ABQ) = \text{ar}(\Delta BQR) + \text{ar}(\Delta PBQ)$

$\text{ar}(\Delta AQC) = \text{ar}(\Delta PBR)$. **1**

28. Since, perpendicular from the centre of the circle to a chord bisects the chord.

$\therefore$ P and Q are the mid-points of AB and CD



$$AP = \frac{1}{2}AB = \frac{1}{2} \times 6 = 3 \text{ cm}$$

$$CQ = \frac{1}{2}CD = \frac{1}{2} \times 8 = 4 \text{ cm}$$ **1**

In right triangle OAP

$$OA^2 = OP^2 + AP^2$$
$$5^2 = OP^2 + 3^2$$
$$OP^2 = 25 - 9$$ **1**
$$OP^2 = 16$$
$$OP = 4 \text{ cm}$$

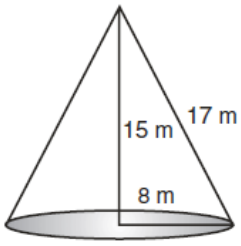
In right ΔOCQ

$$OC^2 = OQ^2 + CQ^2$$
$$5^2 = OQ^2 + 4^2$$
$$OQ^2 = 25 - 16$$ **1**
$$OQ^2 = 9$$
$$OQ = 3 \text{ cm}$$

$\therefore$ $PQ = OP + OQ = 4 + 3 =$ **7 cm** **1**

29. Height of conical tent $(h) = 15$ m

Radius $(r) = \frac{16}{2} = 8$ m



Slant height $(l) = \sqrt{h^2 + r^2}$

$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$

$= \sqrt{289} = 17$ m **1**

Area of canvas = surface area of conical tent **1**

$$l \times b = \pi r l$$
$$l \times 2 = 3{\cdot}14 \times 8 \times 17$$
$$l = \frac{3{\cdot}14 \times 8 \times 17}{2} = \textbf{213·52 m}$$ **2**

30. (i) Height of can $(h) = 1{\cdot}4$ m $= 140$ cm

Radius $(r) = 0{\cdot}4$ m $= 40$ cm

Volume of can $= \pi r^2 h = \frac{22}{7} \times 40 \times 40 \times 140$

$= 2 \times 352000 \text{ cm}^3 = 704000 \text{ cm}^3$ **1**

Volume of milk $= \frac{3}{4} \times$ volume of cylindrical can

$= \frac{3}{4} \times 704000 =$ **528000 cm^3** ½

This milk is poured into some small cylindrical glasses whose height is 10 cm and radius 5 cm.

$\therefore$ No. of small glasses

$$= \frac{\text{Volume of milk}}{\text{Volume of one small glass}}$$ **1**

$$= \frac{528000}{\frac{22}{7} \times 5 \times 5 \times 10} = \frac{528000 \times 7}{22 \times 5 \times 5 \times 10}$$

$= \textbf{672.}$ **1½**

31. **(i)** For Histogram,

Y-axis = one square = one students

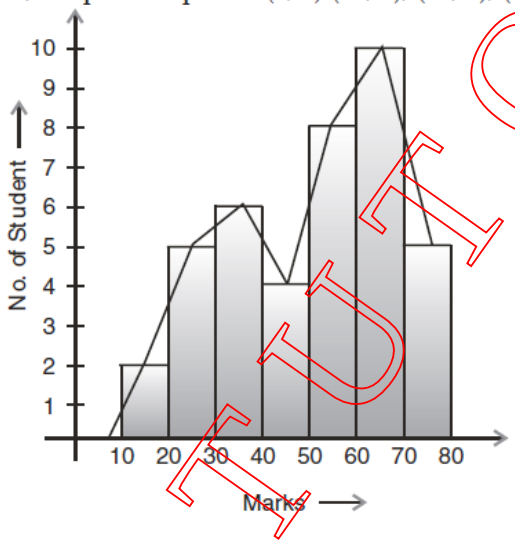
X-axis = one square = 10 marks

(ii) For frequency polygon, first we obtain the class marks. ½

Marks	Class Marks	No. of students
0 — 10	5	0
10 — 20	15	2
20 — 30	25	5
30 — 40	35	6
40 — 50	45	4
50 — 60	55	8
60 — 70	65	10
70 — 80	75	5

1

To obtain the frequency polygon, we plot the points (5, 0) (15, 2), (25, 5), (35, 6), (45, 4) (55, 8), (65, 10), (75, 5)



1½

(i) Statistics. ½

(ii) Sincerity ½